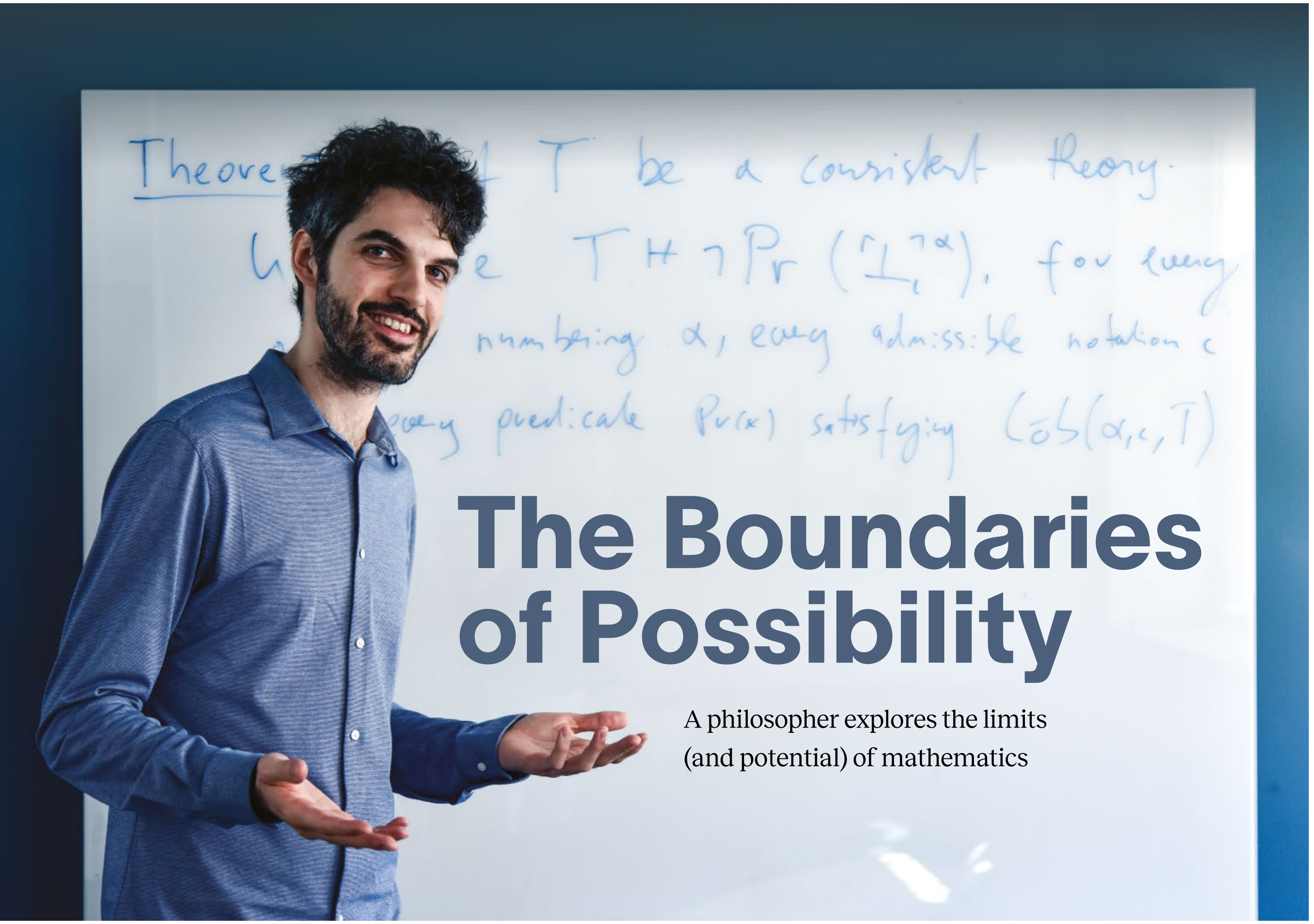

The Boundaries of Possibility

A philosopher explores the limits (and potential) of mathematics

"Why are mathematical truths somehow eternal?" asks Balthasar Grabmayr, who, thanks in part to his Azrieli Fellowship, is starting a new position this spring as a junior professor of philosophy at Germany's University of Tübingen. "Once we prove a theorem, it will hold for eternity. How is that possible? *That* I find super fascinating. It seems like magic."

By Dan Falk
Photograph by Boaz Perlstein

Four centuries ago, Galileo famously described the physical world as a realm that was rooted in mathematics. The universe, he wrote, "cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."

Since Galileo's time, scientists and philosophers have continued to ponder the question of why mathematics is so shockingly effective at describing physical phenomena. No one would deny that this is a deep question, but for philosopher Balthasar Grabmayr, an Azrieli International Postdoctoral Fellow at the University of Haifa, even deeper questions lie beneath it. Why does mathematics work at all? Does mathematics have limits? And if it does, what can we say about those limits?

"I am really fascinated with foundational questions about mathematics," says Grabmayr, who completed master's and undergraduate degrees in math, earned a PhD in philosophy from Humboldt University of Berlin, did postdoctoral research in computer science at Tel Aviv University in 2021, and is now working at the intersection of these three areas. "Philosophical questions such as, 'What is a number? What can mathematics reduce to?' As opposed to many of my peers, who were interested in applications in physics and technology, I was really more interested in this philosophical background."

These questions may sound pie-in-the-sky, which would be par for the course in philosophy. But as elemental as they are, they may also have practical significance, perhaps leading to a better understanding of how computation works and what its limits are. This line of inquiry could also shed light on the nature of language, as well as long-standing puzzles about the functioning of the mind.

Grabmayr found his way to this field from a very different passion: music. Growing up in Vienna, he attended a music conservatory and was set on becoming a classical musician. Eventually, he began to think about what made music work, and then began to think about musical structure. "I started to realize that, actually, what I'm interested in — what I found so attractive in music — is basically mathematics," he recalls. "Mathematics is the science of structure. I was completely captured by that."

He was captivated, too, by the vast sweep of mathematics: once you discover a mathematical truth, it appears to be true everywhere, and for all time. Schoolchildren learn equations like the quadratic formula unquestioningly, but for Grabmayr, such seemingly simple truths contain within them a world of mystery. "Why are mathematical truths somehow eternal?" he muses. "Once we prove a theorem, it will hold for eternity. How is that possible? *That* I find super fascinating. It seems like magic."

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One of Grabmayr's main areas of research involves Gödel coding, a technique that, roughly put, allows mathematics to study itself. Gödel coding lets you convert statements about a system of rules or axioms into statements within the original system.

As Grabmayr explains, it's a way to study not the trees of mathematics but the forest. Take mathematical proofs, for example. "Usually, we use proofs in mathematics to establish a result about numbers, or about groups, or about geometric objects," he explains. But to study proofs in their own right, "we have to change the perspective — now the objects of investigations are proofs and our mathematical theories themselves. So suddenly we take a step out of the usual mathematical framework. Now we're looking at mathematical theories and adding mathematical reasoning from outside."

Gödel coding is named for the Austrian logician Kurt Gödel, who in the 1930s developed his famous "incompleteness theorems," which point to the inherent limitations of mathematics. Although expressed as an equation, Gödel's proof was based on the idea that a sentence such as "This statement is unprovable" is both true and unprovable. As Rebecca Goldstein's biography of Gödel declares, he "demonstrated that in every formal system of arithmetic there are true statements that nevertheless cannot be proved. The result was an upheaval that spread far beyond mathematics, challenging conceptions of the nature of the mind."

Grabmayr's work builds on the program that Gödel began nearly a century ago. "What I'm really interested in is what the limitations of mathematics are," he says. "What are the limits of what we can prove? What are the limits of what we can express in formal languages? And what are the limits of what we can calculate using computers?" (That last remark shows that Gödel coding is of interest well beyond the philosophy of mathematics. "We're surrounded by it," says Grabmayr. "I mean, without Gödel coding there wouldn't be any computers.")

Another potential application is in cognitive science and the study of the mind. Psychologists and other scientists have long debated to what extent the mind is, or is not, like a computer. When we "think," are we manipulating symbols the way a computer does? The jury is still out on that question, but Grabmayr believes his work can at least point toward some answers. "Cognitive science is based on the premise that we can use computational models to capture certain phenomena of the brain," he says. "Artificial intelligence, also, is very much concerned with trying to formally capture our reasoning, our thinking processes."

Much of Grabmayr's current research focuses on this question of whether, or to what extent, the mind can be represented through the formal systems used by computer scientists — a line of inquiry that could influence research in AI and the quest to build artificial minds. His work largely entails reading, constructing and rejecting proofs — "my job consists of making one mistake after the other," he says — by writing mathematical formulas on paper, or on blackboards when he is collaborating.

Albert Visser, a philosopher and logician at Utrecht University in the Netherlands and one of Grabmayr's PhD supervisors, sees a number of potential payoffs for this research. "Balthasar's work has some overspill to computer science and linguistics, since it involves a systematic reflection both on coding and on the nature of syntax," he says. "The discussion of ideas from computer science and linguistics in Balthasar's work is also beneficial in the other direction. It informs logicians of the existence and the importance of such ideas."

Grabmayr points out that the path from foundational work to tangible technological benefits can be long and circuitous. "Kurt Gödel and Alan Turing started out discussing or thinking about foundational issues in mathematics, and then, in passing, they invented computers," he says. "That's the whole point of true foundational research: there can be very real-world applications, but they're completely unexpected and cannot be foreseen when the actual work is done."

Meanwhile, he understands that his work is often baffling to non-specialists, a situation he's striving to change. Grabmayr credits the Azrieli Fellowship with allowing him to "focus entirely on my work, and to meet colleagues around the world to discuss new ideas and projects." Recently, he's given presentations to scholars outside his own field. "It's a matter of how to present the material so as not scare them away immediately, to kind of invite them into the conversation," he says. "That's actually a big part of what I'm trying to do now: finding ways to speak about my research in a more accessible way." ▲●■